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311. Proposed by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

Find, by Cardan's Method, the real root (4) of $x^3 - 6x^2 + 10x = 8$.

Solution by G. I. HOPKINS, Instructor in Mathematics and Astronomy, High School, Manchester, N. H.

Substitute $y+2$ for x . Then $y^3 - 2y - 4 = 0$.

Substitute $v+z$ for y . Then, from the well known formula,

$$v = \sqrt[3]{2 + \frac{10}{9}\sqrt{3}} \text{ and } z = \sqrt[3]{2 - \frac{10}{9}\sqrt{3}}.$$

$\therefore 2 + \frac{10}{9}\sqrt{3}$ must be the cube of a binomial, the first term of which is 1, and the second term contains $\sqrt{3}$.

Assume $(1+a\sqrt{3})^3 = 2 + \frac{10}{9}\sqrt{3}$; whence $1+3a\sqrt{3}+9a^2+3a^3\sqrt{3} = 2 + \frac{10}{9}\sqrt{3}$, or $3a(1+a^2)\sqrt{3}+9a^2 = 1 + \frac{10}{9}\sqrt{3}$.

$$\therefore 9a^2 = 1; \text{ whence } a = \frac{1}{3}.$$

$\therefore 1 + \frac{1}{3}\sqrt{3}$ is the cube root of $(2 + \frac{10}{9}\sqrt{3})$.

$\therefore v = 1 + \frac{1}{3}\sqrt{3}$ and $z = 1 - \frac{1}{3}\sqrt{3}$; $v+z = 2 = y$; and, therefore, $x = y+2 = 4$. Also by quadratics as follows:

Multiplying by x , $x^4 - 6x^3 + 10x^2 - 8x = 0$.

$$x^4 - 6x^3 + 9x^2 + x^2 - 8x = 0,$$

$$(x^2 - 3x)^2 + x^2 - 3x - 5x = 0,$$

$$(x^2 - 3x)^2 + (x^2 - 3x) = 5x.$$

$$\text{Adding } x^2 - 3x, (x^2 - 3x)^2 + 2(x^2 - 3x) = x^2 + 2x.$$

$$\text{Adding 1, } (x^2 - 3x)^2 + 2(x^2 - 3x) + 1 = x^2 + 2x + 1.$$

$$\therefore x^2 - 3x + 1 = \pm (x + 1) \text{ and } x = 4.$$

Also solved by J. Scheffer, G. B. M. Zerr, G. W. Hartwell, and a student in Olivet College.

CALCULUS.

268. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

Determine $\phi(y)$, independent of u , so that the equation $\int_0^u (u-y)^{(2p-1)/2} \phi(y) dy = u^m$ is satisfied, p and m being positive integers and $m > p$. Do you notice properties of special interest for any special cases?

Solution by the PROPOSER.

The problem may be generalized with small increase in the difficulty of solution; thus:

It is required to find $\phi(y)$ a function of y alone from the equation

$$(1) \quad f(u) = \int_0^u h(u, y) u^a \phi(y) dy,$$

where $h(u, y)$ is a homogeneous function of u, y of degree zero such that it may be expressed as a finite or an infinite series,

$$h(u, y) = \beta_0 + \beta_1 \frac{y}{u} + \beta_2 \frac{y^2}{u^2} + \beta_3 \frac{y^3}{u^3} + \dots,$$

and where $f(u)$ is a given function which may be written in the form

$$f(u) = c_1 u^{a_1} + c_2 u^{a_2} + c_3 u^{a_3} + \dots, \quad a_1 < a_2 < a_3 < \dots;$$

a 's being positive or negative, entire or fractional, subject to the given relations.

First consider the special case

$$(2) \quad c_i u^{a_i} = \int_0^u (\beta_0 + \beta_1 \frac{y}{u} + \beta_2 \frac{y^2}{u^2} + \dots) u^a \phi_i(y) dy.$$

Now

$$\begin{aligned} & \int_0^u (\beta_0 + \beta_1 \frac{y}{u} + \beta_2 \frac{y^2}{u^2} + \dots) \delta_i y^{a_i - a - 1} dy \\ &= \delta_i u^a \left(\frac{\beta_0}{a_i - a} y^{a_i - a} + \frac{\beta_1}{a_i - a + 1} \frac{y^{a_i - a + 1}}{u} + \frac{\beta_2}{a_i - a + 2} \frac{y^{a_i - a + 2}}{u^2} + \dots \right) \Big|_0^u \end{aligned}$$

$$(3) \quad = \delta_i s_i u^{a_i}, \text{ where}$$

$$(4) \quad s_i = \frac{\beta_0}{a_i - a} + \frac{\beta_1}{a_i - a + 1} + \frac{\beta_2}{a_i - a + 2} + \frac{\beta_3}{a_i - a + 3} + \dots$$

Equating the first member of (2) with the second of (3), we have

$$\delta_i = \frac{c_i}{s_i}. \quad \therefore \phi_i(y) = \frac{c_i}{s_i} y^{a_i - a + 1}.$$

Evidently, then, a solution of (1) is

$$\phi(y) = \sum_i \phi_i(y); \text{ or } \phi(y) = \sum_i \frac{c_i}{s_i} x^{a_i - a - 1},$$

where i runs over all the subscripts of c in the expression for $f(u)$ and s_i is defined by equation (4).

NOTE.—Volterra has studied a more general problem. See *Encyklop. d. Math. Wissenschaft.*, II, p. 808. The problem in the present form affords the solution of several problems in practical hydrodynamics.